UNIVERSITY OF WATERLOO
FACULTY OF ENGINEERING
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ECE 204 *Numerical methods*

Approximating the value of an expression

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Introduction

- In this topic, we will
  - Describe what will be covered in the next topic
    - Evaluating a polynomial
    - Given periodic samples of of an actual function:
      - Estimate the value at a point
      - Estimate the derivative at a point
      - Estimate the integral over an interval
  - Introduce the idea that these samples may either be exact or samples that are combined with random errors
  - Describe how we will use different approaches in each case

Approximating the value of an expression

- We start with approximating the value of a polynomial at a point
- For the next three topics, we are sampling an actual value in either space or time represented by $f(x)$ and $y(t)$
  - We will sample these functions at equally spaced points

    $$x_k = x_0 + kh \text{ or } t_k = t_0 + k\Delta t$$

  - If we are describing the exact value of the functions, we will write it as

    $$(x_k, f(x_k)) \text{ or } (t_k, y(t_k))$$

  - If we are sampling the value with a random error, we will write it as

    $$(x_k, f_k) \text{ or } (t_k, y_k)$$

  - Thus, $f_k \approx f(x_k)$ and $y_k \approx y(t_k)$

Approximating the value of an expression

- Given exact values $(x_k, f(x_k))$ or $(t_k, y(t_k))$, we will
  - Approximate the value of an interpolating polynomial a point
  - Approximating the derivative of an expression at a point
  - Approximating a definite integral

- Exact values may be used when we run simulations:
  - We have a function y that describes the potential difference across a battery over time
- Exact values may be the result of a solution to a simple differential equation with initial- or boundary-values

Approximating the value of an expression

- Next, however, we will assume that the samples have significant random error in the samples f_k or y_k values
  - The previous approximations will be useless
  - We will expand on linear algebra and introduce the concept of least-squares best-fitting linear regression
  - The least-squares best-fitting linear or quadratic polynomials will significantly aid us in estimating the underlying values

Summary

- Following this topic, you now
  - Have an overview of the ideas to be covered in this topic
  - Understand that functions representing actual values that are being sampled either in space or time: $f(x)$ or $y(t)$
  - Are aware we can only sample them periodically:

    $$x_k = x_0 + kh \text{ or } t_k = t_0 + k\Delta t$$
  - Understand that the samples may either be
    - Exact: $f(x_k)$ or $y(t_k)$
    - Combined with error: f_k or y_k
  - Are aware that we will try to approximate the value of polynomials interpolating these points, the derivatives at these points, and integrals over a sequence of these points
  - Understand the goal is to approximate as closely as possible these properties of the actual functions $f(x)$ *and* $y(t)$

References

[1] https://en.wikipedia.org/wiki/Numerical_analysis

Acknowledgments

Tazik Shahjahan for pointing out typos.

Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using `Consolas`. Mathematical equations are prepared in MathType by Design Science, Inc.

Examples may be formulated and checked using Maple by Maplesoft, Inc.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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